Paths of interactive cracks in creep conditions

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ABSTRACT. The paper contains plane strain analysis of uniformly stretched plate working in creep condition. The plate contains initial defects in forms of central and/or edge cracks working in mode I. These cracks are modelled by attributing critical value of damage parameter to preset points and therefore resulting in stresses set to zero (material does not support any loading). The Continuum Damage Mechanics constitutive equations are used to describe the creep crack growth problem and Finite Element Method Abaqus system is applied to solve corresponding boundary and initial value problem. Analysis of different initial cracks configuration has been performed. The crack path is defined by points in which damage parameter equals to critical one. Time to failure of the plate with single initial crack is achieved when the crack path spans its width. This time is calculated and compared to the time to failure of initially uncracked structure. For the plate with multiple cracks the paths starting from different cracks can develop independently until they merge and/or span the plate width. In each case the damage field is analysed and the direction of crack path development is determined. The analysis of crack propagation allows for determination of a distance between initial cracks for which the interaction between them is negligible. It is demonstrated that Continuum Damage Mechanics approach allows not only to model the development of initially existing cracks but also initiation of new, cross-spanning cracks and their kinking and branching.

KEYWORDS. Creep damage; Crack paths; Cracks interaction; Integrity assessment.

INTRODUCTION

Evaluation of remaining life time of a structure working in creep condition is important engineering problem. It is especially crucial if some initial flaws exist what may influence the cracks and their pattern development causing premature failure of the structure. For the structure working in elastic regime, the flaws can be modelled by the cracks and the structure can be analysed by Linear Fracture Mechanics (LFM) tools. Each single crack can be modelled independently and critical length of it can be evaluated. Much more complicated is the problem of cracks interaction. If distance between neighbouring cracks is comparable with the length of the cracks, they should be analysed together. This problem was analysed by many authors (e.g. [4, 5, 6, 9, 10, 12]). In frame of LFM the interaction is expressed in terms of Stress Intensity Factor (SIF). The cracks interaction can cause amplification of SIF or its decreasing. The former effect occurs mainly for coplanar cracks, the latter for parallel cracks due to shielding effect. In both cases the interacting cracks can be replaced by single substitute crack which size depends on dimensions and relative position of original cracks (cf. e.g. [10]).

If the structure works at elevated temperature, that is in creep conditions, the analysis using SIF is limited to a few cases. It can be applied if the stress redistribution from elastic state to steady creep state is small. For steady state creep condition the C*-integral (analogous to J-integral) can be used (cf. e.g. [14, 15, 17]). Thanks to this parameter different configurations of cracks can be compared and parameters of single equivalent crack can be determined. But in this method the steady state stress field is considered as constant and the stress redistribution due to damage development is not taken into account. Another method is that by means of Continuum Damage Mechanics. It allows for tracing stress and damage history, and to determine the most probable crack development. This methodology was successfully applied to creep growth of single crack by [7, 13] and is used by authors in this paper to determine the paths of interacting multiple cracks in creep conditions.

CREEP VERSUS ELASTIC SAFETY OF STRUCTURES

For perfect elastic structures its loading capacity can be evaluated basing on two limit cases: ultimate strength for uncracked members or critical load to propagate existing defects. The first one is standard procedure for newly projected structure, the second one - for structures which contain one or multiple cracks.

In creep conditions, however, which has to be taken into account when structures is expected to work in a severe environmental conditions (e.g. high temperature or chemically aggressive media), the concept of critical loading must be replaced by the notion of critical time to failure. The latter is an effect of material structure deterioration which occurs at any level of mechanical load. This is similar to the effect of sloping of both part of Wöhler diagrams and fatigue limit absence in high temperature applications.

In the case of appearance of multiple linear defects (cracks) a new approach has to be developed because of interaction between individual cracks. In elasticity it requires solving a troublesome individual BVP (Boundary Value Problem) for each initial crack configuration. This procedure can be neglected when cracks interaction is weak, and critical load can be calculated basing on the SIF value for most dangerous crack.

The assumption of nonexistence of cracks interaction can not hold in creep conditions. The material deterioration will grow for any load, even for initially flawless structures, causing the slowly movement of cracks which will always grow - albeit not necessarily dangerous - and may coalescence with catastrophic result.

In the present paper the methodology of dealing with such a situation will be developed and illustrated by an simple example of a plane rectangular specimen subjected to uniaxial tension with two symmetric edge cracks and/or a central one.

CONSTITUTIVE EQUATIONS USED AND NUMERICAL METHODOLOGY

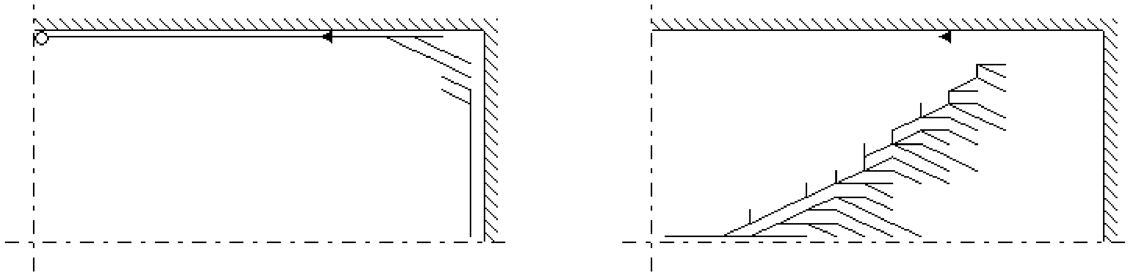
Analytical modelling of material deterioration in creep conditions became possible due to L.M. Kachanov works initiated in late 50-ties of last century [8]. When damage is coupled with stress-strain rate equation (cf. [2]) it yields a set of differential nonlinear equations:

$$\varepsilon_{ij} = D^{-1}_{ijkl}\,\sigma_{kl} + \varepsilon^{c}_{ij}, \tag{1a}$$

$$\frac{\partial \varepsilon^{c}_{ij}}{\partial t} = \gamma \left(\frac{\sigma_{eff}}{1-\omega} \right)^{n} \frac{\partial \sigma_{eff}}{\partial \sigma_{ij}}, \tag{1b}$$

$$\frac{\partial \omega}{\partial t} = A\left[\alpha \frac{\sigma_{\max}}{1-\omega} + (1-\alpha)\frac{\sigma_{eff}}{1-\omega} \right]^{\mu}, \tag{1c}$$

where ε_{ij}, ε^{c}_{ij} are total and creep strain tensors, σ_{ij} is stress tensor, D_{ijkl} is elastic constants matrix, γ, n, A, μ, α are steady-state creep and damage material constants, σ_{max}, σ_{eff} - main positive principal stress and Huber-Mises effective stress, ω - scalar damage parameter ($0 \le \omega \le 1$), t - time. This approach has been used in several investigations [1, 2, 3] and presented also at the series of conferences “Crack Paths”. The picture below demonstrates effectiveness of this approach used in analysis of crack paths in 3D bending plates [3]:



(a) (b)

Figure 1: Crack patterns on upper (a) and lower (b) surfaces of a 3D plate without initial damage.

The above methodology allows for evaluation all three characteristic stages of damage growth terminated by: t_1 - time when first failure occurs in a point, t_2 - time when failure occurs across of structures characteristic length, t_3 - time when the cracks system makes structure unstable.
It is worthwhile to notice the 'Kachanov paradox', which follows his approach, that for uniform stress distribution (uniaxial tension) the damage in all points grows simultaneously causing the specimen to "evaporate" at time $t_1 = t_2 = t_3$!
This is not the case when stress field in a structure is non-homogeneous one - like in plates, for example - and which is particularly strong in the case of initial discontinuities (cracks) existing in a structure.
Because of its nonlinearity and complexity the problems considered below will be solved by means of numerical analysis. Time integration of Eq. (1) is done by Euler explicit method. Plane strain four- and three-node solid elements are used. A diversification of a FE mesh has been made in damage affected zones. The numerical solution of the problem is very strong mesh depended (cf. e.g. [14]), so the results can be considered as qualitative ones.
The initial cracks are defined as the set of Gauss integration points in which initial value of damage parameter ω=1 (failure in a point).

SPECIMEN GEOMETRY, LOADING AND MATERIAL CHARACTERISTICS

A rectangular plate of dimensions shown in Fig. 2a is subjected to uniformly distributed loading q on its end edges. The plane strain condition is assumed and the problem is solved as 2D one, the times t_2 and t_3 are equal.
The following configurations of initial crack will be considered: two symmetric edge cracks (Fig. 2b), two symmetric central cracks (Fig. 2c), and combination of two previous configurations (Fig. 2d), where the distance denoted as H will be used as a problem free parameter. Only quarter part (right upper) of the plate is modelled due to symmetry. Unlikely to elastic considerations, for creep conditions an interaction between multiple cracks will always exists as damage field extends over the whole structure. The critical distance will be sought for which the interaction of edge and central cracks can be neglected (it is when they will span plate width independently). It is necessary to emphasise that this critical distance has to be understood as that corresponding to the situation when cracks which span specimen edges at time t_2 will be different form cases 2c and 2d by a small, arbitrarily defined amount.
The material parameters used for simulation was that of type 316 stainless steel at 650°C according to [11]: $E=0.144\cdot 10^6$MPa, $\nu=0.314$, $\gamma=2.13\cdot 10^{-13}(\mathrm{MPa})^{-n}$ h^{-1}, $n=3.5$, $A=3.42\cdot 10^{-9}(\mathrm{MPa})^{-\mu}$ h^{-1}, $\mu=2.8$, $\alpha=1.0$. The loading was assumed to be 50 MPa and is kept constant for all experiments.
For the structure shown in Fig. 2b the length of initial edge crack is assumed to be l_e=15 mm. SIF for this crack and assumed loading is 12.3 MPam$^{0.5}$. This value is about one order less than critical value of SIF, so the structure is safe in elastic conditions. For the structure shown in Fig. 2c the length of initial central cracks are assumed to yield the same critical elastic load as in the case of central crack and equals to 18 mm (SIF is equal to 12.3 MPam$^{0.5}$, too).
For the initially non-cracked structure (Fig. 2a) the time to failure can be calculated independent of deformation since the stress σ_x (the only nonzero- component of stress matrix) is - from equilibrium - equal to loading q. Simple integration of the Eq. (1c) in uniaxial stress state with initial condition ω=0 and failure at ω=1 yields:

$$t_0 = \frac{1}{(\mu+1)Aq^{\mu}} = 5115 \text{ h}. \quad (2)$$

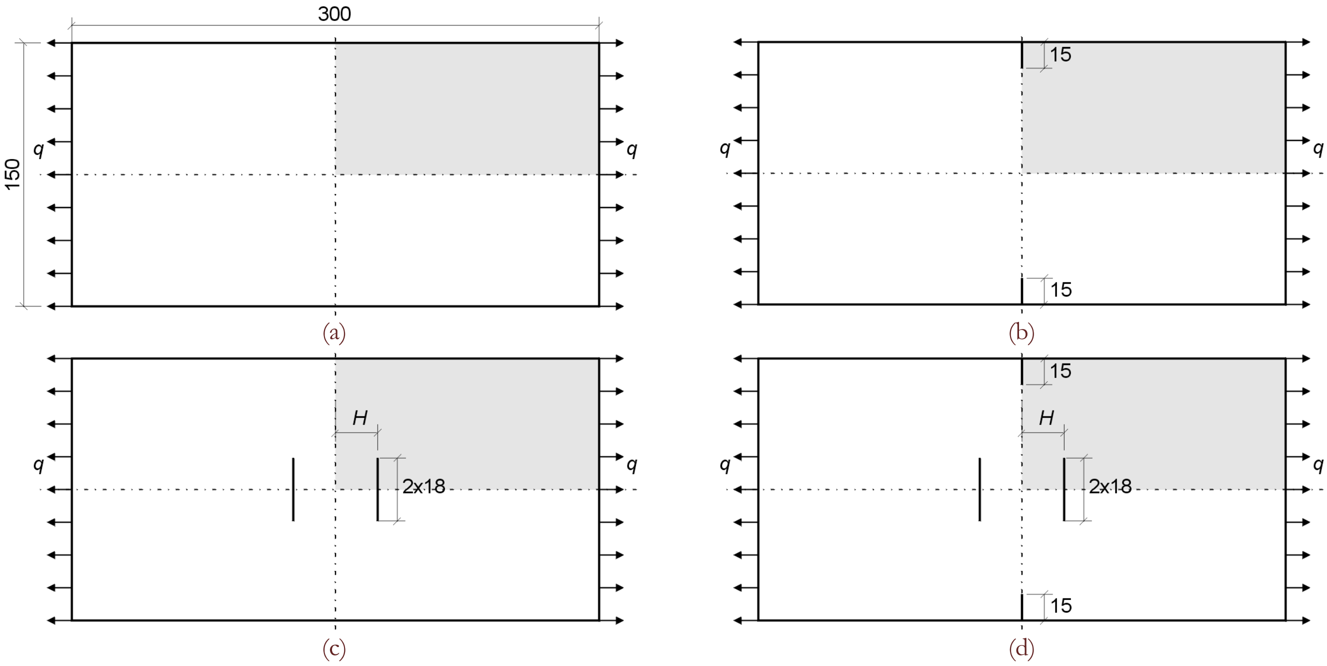


Figure 2: Initial uncracked structure (a), structure with edges cracks (b), with central cracks (c) and with both cracks types (d). Dimensions are in millimetres. Due to symmetry the calculations are performed in the greyish quadrant of the plate.

ANALYSIS OF A PLATE WITH SINGLE INITIAL CRACK

Edge crack

For the structure with edge crack (Fig. 2b) the time of first failure occurrence is found to be t_{1e}=99.1h. Then the crack develops due to damage growth and finally spans the width of the plate in time t_{2e}=795h. As the damage is driven by the maximum principal stress (α=1.0) the path of the crack is straight. The rate of the crack growth at the beginning is equal to 1.84E-5m/h and it is increasing as the crack becomes longer. When the crack length achieves its critical value in creep condition the rate of the growth becomes relatively high and is equal to 1.22E-2m/h (marked in Fig. 3 by a red point). The critical length of the crack at this instant is close to 68mm.

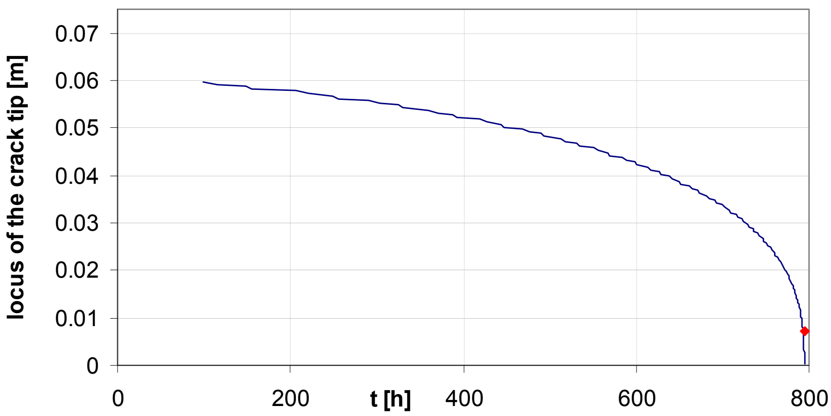


Figure 3: Development in time of the edge crack.

Small variations of the curve shown in Fig. 3 is because the crack growth rate is subjected to small fluctuations as the constitutive model consists of two parts, elastic and creep ones. The elastic stress is very high at crack tip and due to relaxation this high value is decreasing in time. Simultaneously the damage is developing, the crack tip changes its position and new elastic stress is raised.

Single central crack

The next structure analysed is that with a single central crack (parameter H=0 in Fig. 2c). The time of first damage occurrence and ultimate time to failure are: t_{1c}=94.1 h and t_{2c}=715 h. The differences to the previous case are very small - 5% and 10% respectively. The critical length of the crack is 67mm, so it is also very close to critical length of edge crack.

PLATE WITH TWO CENTRAL CRACKS

The structure with two central cracks (Fig. 2c) exhibits the presence of shielding effect. In elastic regime for infinite plate the SIF is going to SIF of single crack when H is going to infinity (c.f. e.g. [9, 16]); for smaller value of H the structure becomes more safe due to shielding. The analysed values of parameter H are from 1 to 25mm with step 1mm for H<5mm, and 5mm - for larger H. The cracks in first two cases (H=1mm and H=2mm) are joining together producing single central crack (see Fig. 4). The structure becomes unstable due to cutting off the central part. Obtained times to failure are of the order of time to failure of single central crack t_{2c} (see Fig. 5). The intermediate cases (H from 3 to 15mm) produce straight cracks and greater times to failure (about 800h). For the larger distance (H>15mm) the times to failure are smaller and do not differ much from the time to failure of single central crack t_{2c}. So the shielding effect is the most pronounced for H in its medium range. For larger distance the long term behaviour is similar to behaviour of single crack so crack interaction can be dealt as negligible. Slightly different situation is for time of first failure occurrence t_1 (see Fig. 6). Except the case H=0mm the time t_1 monotonically decreases with growing H. This situation corresponds to growing SIF in elastic regime as the time t_1 depends partly on elastic stress distribution in contrast to the time t_2. The critical length of crack, defined as the length of crack with rate growth going to infinity, is almost constant and is equal to about 67mm (see Fig. 7).

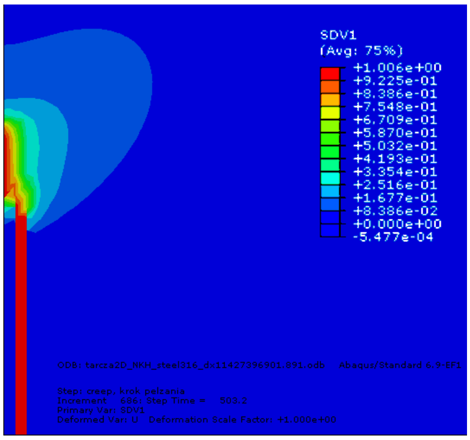


Figure 4: Damage distribution close to final failure for the quarter central part of the plate for central crack H=1mm.

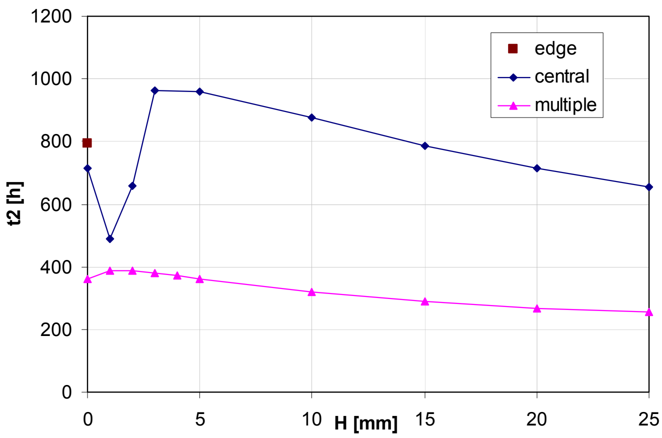


Figure 5: Time to failure.

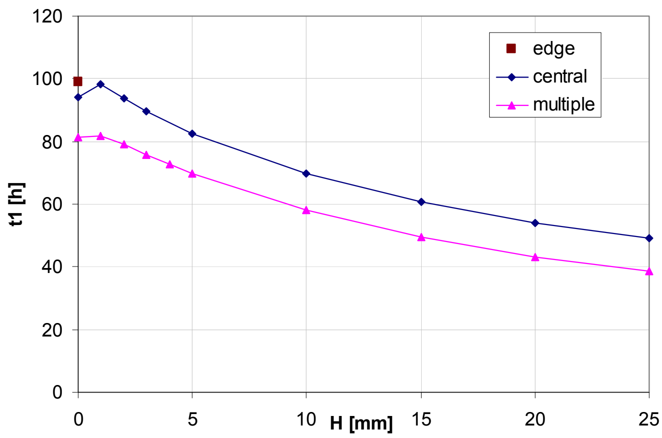


Figure 6. Time to first failure occurrence.

PLATE WITH MULTIPLE INITIAL CRACKS

The multiple crack configuration consists of two edge cracks and two central ones (see Fig. 2d). The numerical analysis is performed for values of H parameter from 0 to 25mm. The crack paths for all examined configurations are shown in Fig. 8. It can be noticed that for small values of distance H, the crack remains straight until the central and edge cracks meet together, and kink to join. For larger values of H, the cracks are bypassing each other, but damage field develops in area between cracks, causing them to join together. In some situations a new crack arises, transverse to main ones. For large values of H the central crack keeps developing in its main direction, as the edge crack stops its straightforward growth and becomes the transversal crack. The area behind the central crack is not stretched and value of damage parameter is close to 0, all the time. The interactions of both cracks is always visible in all examined configurations (also for time t_1). So the distance H of not interacting cracks should be greater than 25 mm. This is in contradiction to commonly used 'rule of thumb' in elastic conditions, that for distance greater than crack length (18 mm) the interaction of cracks is negligible.

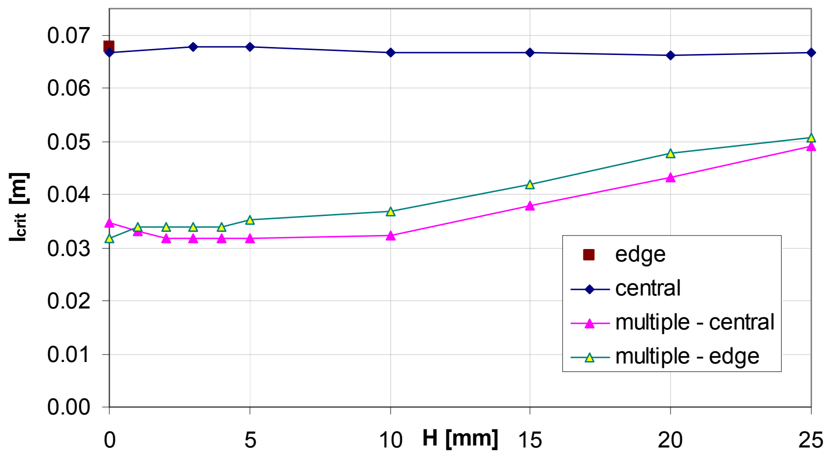


Figure 7. Critical length of cracks.

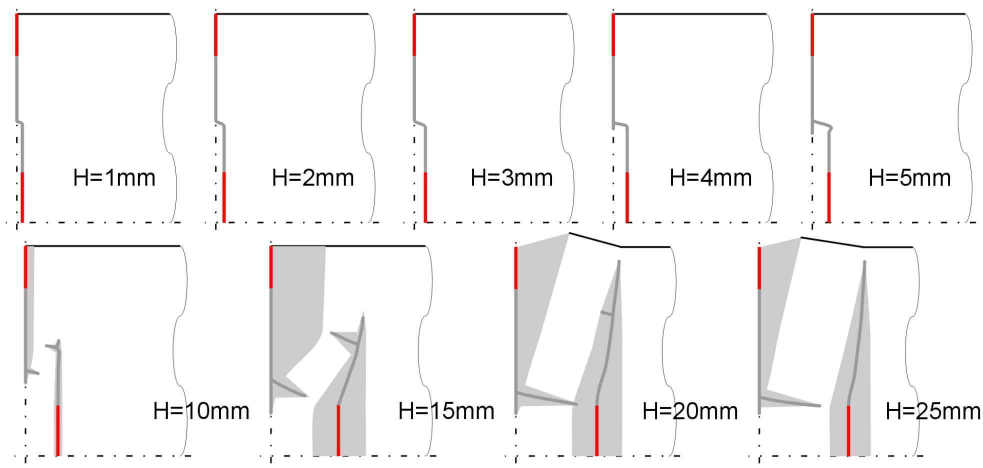


Figure 8. Crack paths for multiple cracks configurations. Red are initial cracks, dark grey - crack at time to failure, light grey - crack opening area.

Comparison of times to failure (Fig. 5) for cases 2c and 2d shows that the edge crack in all configurations shortens significantly the specimen life (except the H=1 and 2 mm cases, but here another failure mechanism determines the time t_2 for central crack configuration). The reduction in time is close to 60%. It is much larger than reduction of time t_1 which is up to 20% (Fig. 6). It can be concluded that the evolution of damage field in time multiplies the effect of cracks interaction.

Analysis of critical lengths of cracks (Fig. 7) shows that the final crack growth acceleration for small values of H is only when the cracks meet together - the calculated critical length is close to half of the plate width. For larger values of H this length is not constant. Therefore, it is difficult to propose any formula for equivalent length. Thus in time dependent analysis and for multiple crack configurations the failure criterion based on critical length of the crack is meaningless.

FINAL REMARKS

The solutions obtained demonstrate ability of adopted methodology to show creep crack paths in the case of existence of interactive initial cracks. Different cracks behaviour like its merging, kinking and branching become possible. The next step one should consider influence of geometry of structure (width and length of a specimen under tension) as well as the length of initial cracks making this analysis useful in engineering practice.

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